Promociona 8 (ocho)

Apellido y Nombres: ....
Dni: .... Comisión: 57206 ....

ANALISIS (72) 2^{do} PARCIAL 2C2024 C Tema 4

1	2	3	4	NOTA
B	B	B	R	8 (ocho)

1. Calcular $\lim\limits_{x \to 0} \dfrac{e^{4x} + e^{-4x} - 2}{3x^2}$.

2. Calcular $\displaystyle\int x\sqrt{x+2}\, dx$.

3. Hallar el área de la región encerrada por el gráfico de $f(x) = -\dfrac{5}{x} + 6$ y la recta $y = x$.

4. Decidir si la serie $\displaystyle\sum_{n=0}^{\infty} 2^{-2n+1}$ es convergente y, en caso de serlo, calcular su suma.

1 $\lim_{x\to 0} \dfrac{e^{4x}+e^{-4x}-2}{3x^2} = \dfrac{0}{0} \to \underline{L'H}$

$\lim_{x\to 0} \dfrac{4e^{4x}+(-4)e^{-4x}}{6x} = \dfrac{0}{0} \to \underline{L'H}$

$\lim_{x\to 0} \dfrac{16e^{4x}+16e^{-4x}}{6} = \dfrac{16\cdot 1+16\cdot 1}{6}$

$= \dfrac{\cancel{32}}{\cancel{6}} = \boxed{\dfrac{16}{3}}$

B

RTA: $\lim_{x\to 0} \dfrac{e^{4x}+e^{-4x}-2}{3x^2} = \dfrac{16}{3}$

CALCULOS AUXILIARES

$(e^{4x})' = 4e^{4x}$

$(e^{-4x})' = -4e^{-4x}$

$(3x^2)' = 6x$

$(4e^{4x})' \to \underbrace{(4)'}_{=0}e^{4x} + 4.(e^{4x})'$

$4.4e^{4x} = \boxed{16e^{4x}}$

$\underbrace{(-4)'}_{=0}e^{-4x} + (-4)(e^{-4x})'$

$(-4)(-4)e^{-4x} = \boxed{16e^{-4x}}$

$(6x)' = 6$

2 $\int \underset{f(x)}{x}\,\underset{g'(x)}{\sqrt{x+2}}\,dx$

$f(x) = x \to f'(x) = 1$

$g'(x) = \sqrt{x+2} \to g(x) = ?$

$\int \sqrt{x+2}\,dx \to \int \sqrt{t}\,dt \to \int t^{1/2}\,dt$

$t = x+2$ $\qquad \dfrac{t^{3/2}}{\frac{3}{2}} = \dfrac{2t^{3/2}}{3} + C \Rightarrow g(x) = \dfrac{2(x+2)^{3/2}}{3}$

$dt = 1\,dx$

$dt = dx$ $\qquad f(x)\cdot g(x) - \int (f'(x)\cdot g(x))\,dx$

$x \cdot \dfrac{2(x+2)^{3/2}}{3} - \int 1 \cdot \dfrac{2(x+2)^{3/2}}{3}\,dx$

$\dfrac{2x(x+2)^{3/2}}{3} - \dfrac{2}{3}\int (x+2)^{3/2}\,dx$

$t = x+2$

$dt = 1\,dx$

$dt = dx$

$\dfrac{2x(x+2)^{3/2}}{3} - \dfrac{2}{3}\int t^{3/2}\,dt$

$\dfrac{2x(x+2)^{3/2}}{3} - \dfrac{2}{3} \cdot \dfrac{t^{5/2}}{\frac{5}{2}} = \dfrac{2x(x+2)^{3/2}}{3} - \dfrac{2}{3} \cdot \dfrac{2(x+2)^{5/2}}{5}$

$\downarrow$

$\dfrac{2t^{5/2}}{5}$ $\qquad \rightarrow$

$$\frac{2x(x+2)^{3/2}}{3} - \frac{2}{3}\cdot\frac{2(x+2)^{5/2}}{5} \Rightarrow \boxed{\frac{2x(x+2)^{3/2}}{3} - \frac{4(x+2)^{5/2}}{15} + C}$$

B

3 $f(x) = -\frac{5}{x} + 6 \mid y = x \rightarrow g(x) = x$

$$\cancel{-\frac{5}{x}+6 = x \rightarrow \ldots}$$

$$\cancel{-\frac{5}{x}+6-x=0 \rightarrow x((-5)^{-1}-1)+6=0}$$

$$\cancel{x((-5)^{-1}-1) = -6}$$

$$\cancel{(-5x)^{-1} - x + 6}$$

$$\cancel{-x(5^{-1}+1)+6=0}$$

$$\cancel{-x(5^{-1}+1) = -6}$$

$$\cancel{-x = \frac{-6}{5^{-1}+1} \rightarrow x = -\left(\frac{-6}{5^{-1}+1}\right) \rightarrow x = \frac{6}{(5^{-1}+1)}}$$

$$\cancel{x = \frac{6}{\frac{1}{5}+1} \rightarrow x = \frac{6}{\frac{1}{5}+\frac{5}{5}} \rightarrow x = \frac{6}{\frac{4}{5}} \rightarrow x = \frac{6\cdot 5}{4}}$$

$$\cancel{x = \frac{30}{4} \rightarrow x = \frac{15}{2}}$$

$$\cancel{-\frac{5}{x}+6 = x}$$

$$\cancel{6 = x + \frac{5}{x}}$$

$f(x) \cap g(x)$

$$-\frac{5}{x}+6 = x$$

$$-\frac{5}{x} = x - 6$$

$$-5 = (x-6)x$$

$$-5 = x^2 - 6x$$

$$0 = x^2 - 6x + 5$$

a, b, c

$$\frac{-b \pm \sqrt{b^2 - 4\cdot a\cdot c}}{2\cdot a}$$

$$\frac{-(-6) \pm \sqrt{(-6)^2 - 4\cdot 1\cdot 5}}{2\cdot 1}$$

$$= \frac{6 \pm \sqrt{36-20}}{2}$$

$$= \frac{6 \pm 4}{2}$$

$$x_1 = \frac{6+4}{2} \Rightarrow x_1 = 5$$

$$x_2 = \frac{6-4}{2} \Rightarrow x_2 = 1$$

$$f(x) \cap g(x) : \{1; 5\}$$

[C.A]

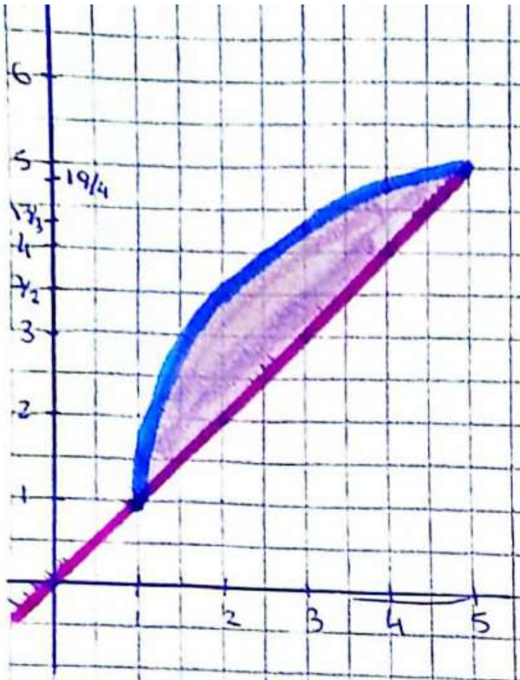
$$-\frac{5}{2}+6 = \frac{-5+12}{2} = \frac{7}{2}$$

$$-\frac{5}{3}+6 = \frac{-5+18}{3} = \frac{13}{3}$$

$$-\frac{5}{4}+6 = \frac{-5+24}{4} = \frac{19}{4}$$

$$-\frac{5}{5}+6 = -1+6 = 5$$

x	f(x)	g(x)
0		
1	1	1
2	7/2	2
3	13/3	3
4	19/4	4
5	5	5



$f(x) \rightarrow$ techo
$g(x) \rightarrow$ piso
$\int_1^5 \left(f(x) - g(x) \right) dx$

Área

$A = \int_1^5 \left(\left(-\frac{5}{x} + 6 \right) - (x) \right) dx$

$A = \int_1^5 \left(-\frac{5}{x} + 6 - x \right) dx$

$A = \int_1^5 \left(-5 \cdot \frac{1}{x} + 6 - x \right) dx$

$A = \left[(-5) \ln(x) + 6x - \frac{x^2}{2} \right|_1^5$

$A = \left((-5) \cdot \ln(5) + 6 \cdot 5 - \frac{5^2}{2} \right) - \left(\cancel{(-5) \ln(1)} + 6 \cdot 1 - \frac{1^2}{2} \right)$

$A = \left(-8{,}04 + 30 - \frac{25}{2} \right) - \left(6 - \frac{1}{2} \right)$

$A = 9{,}46 - 5{,}5$

$\boxed{A = 3{,}96}$

$\boxed{C.A.}$

$\frac{x^2}{2} = \frac{(x^2)' \cdot 2 - \cancel{x^2 \cdot (2)'}}{2^2}$

$\frac{\cancel{2}x^2}{4} = x$

Rta: el área de la región encerrada por las gráficas de $f(x)$ y $g(x)$ es 3,96.

B)

$\frac{1}{4} < 1 \rightarrow \boxed{\text{converge}}$

a

$\boxed{4}$ $\sum_{n=0}^{\infty} 2^{-2n+1} \rightarrow \sum_{n=0}^{\infty} \left(\left(\frac{1}{2} \right)^2 \right)^{n+1} \rightarrow \sum_{n=0}^{\infty} \left(\frac{1}{4} \right)^{n+1} \rightarrow \frac{1}{4} \sum_{n=0}^{\infty} \left(\frac{1}{4} \right)^n$

$\rightarrow \frac{1}{4} \cdot \frac{1}{1 - \frac{1}{4}} = \frac{1}{4} \cdot \frac{1}{\frac{3}{4}} \rightarrow \frac{1}{\cancel{4}} \cdot \frac{\cancel{4}}{3} = \boxed{\frac{1}{3}}$

Rta: $\sum_{n=0}^{\infty} 2^{-2n+1}$ es convergente y $= \frac{1}{3}$.

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